國中第三冊

立即練習五

p. 11 (1)

大半圓的面積：$\dfrac{1}{2} \times x^2 \times \pi = \dfrac{x^2}{2}\pi$

小圓的面積：$\left(\dfrac{x}{2}\right)^2 \times \pi = \dfrac{x^2}{4}\pi$

$\therefore \dfrac{x^2}{2}\pi - \dfrac{x^2}{4}\pi = \dfrac{x^2}{4}\pi$（平方單位）

p. 11(2)

設 $x = 2006$

原式：$(x-1)(x+2) - x(x+1) + (x-3)(x-1) - (x-2)^2$

$\qquad = x^2 + x - 2 - x^2 - x + x^2 - 4x + 3 - x^2 + 4x - 4$

$\qquad = -3$

p. 11(3)

$2^x = 16 \Rightarrow x = 4 \qquad\qquad 3^y = 27 \Rightarrow y = 3$

$(2x+y)(2x-y) = 4x^2 - y^2 = 55$

立即練習六

p. 25(1)

$A = B \times Q + R = aB \times \dfrac{Q}{a} + R \qquad \therefore$ 商式為 $\dfrac{Q}{a}$，餘式為 R

p. 25(2)

$f(x) = (3x+1)(2x^2-1) + (-1) = 6x^3 + 2x^2 - 3x - 2$

$f(-2) = 6(-2)^3 + 2(-2)^2 + 6 - 2 = -36$

p. 25(3)

$\because xy + 7 = 2x - 2y \qquad \therefore xy - 2x + 2y = -7$

$\therefore (x+2)(y-2) = xy - 2x + 2y - 4 = -11$

p. 25(4)

$(5x-1) - (x+3) \times 2 = 3x - 7$

$\therefore (5x-1)^2 - (3x-7)^2 = 16x^2 + 32x - 48$

立即練習四

p. 44(1)

$$2 + \frac{34}{b} = \left(\frac{c}{9}\right)^2 = \frac{c^2}{81} \quad 可知\ b = 81$$

$$, 2 + \frac{34}{81} = \frac{196}{81} = \left(\frac{14}{9}\right)^2 \quad c = 14 \quad \therefore b + c = 81 + 14 = 95$$

p. 44(2)

設小天 a 元　則 $3a^2 = 768 \Rightarrow a = 16$ 或 -16　（-16 不合）

p. 44(3)

$$|2x + 3y - 3| + |3x - 4y - 13| = 0$$

$$\therefore \begin{cases} 2x + 3y - 3 = 0 \\ 3x - 4y - 13 = 0 \end{cases} \Rightarrow \begin{cases} 2x + 3y = 3 \cdots ① \\ 3x - 4y = 13 \cdots ② \end{cases}$$

由 ① × 4 + ② × 3 得 $x = 3$ 代入①得 $y = -1$

$$\therefore 8x - y = 25\ 的平方根爲 \pm \sqrt{25} = \pm 5$$

立即練習三

p. 52(1)

$$① \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2} = 51 - 2 = 49 \quad \therefore x - \frac{1}{x} = \pm 7$$

$$② \sqrt{196 - 2 + \frac{1}{196}} = \sqrt{14^2 - 2 + \left(\frac{1}{14}\right)^2} = \sqrt{\left(14 - \frac{1}{14}\right)^2}$$

$$= 14 - \frac{1}{14} = 13\frac{13}{14}$$

p. 52(2)

$$\sqrt{20} = 4.4 \cdots \quad \therefore \sqrt{20} = 4 + b \Rightarrow 小數部分\ b = \sqrt{20} - 4$$

$$a^2 + b^2 - 2ab = (a - b)^2 = \left[4 - \left(\sqrt{20} - 4\right)\right]^2$$

$$= \left(8 - 2\sqrt{5}\right)^2 = 64 - 32\sqrt{5} + 20 = 84 - 32\sqrt{5}$$

p. 52(3)

① $\dfrac{\sqrt{3}}{\sqrt{5}} = \dfrac{\sqrt{15}}{5}$, $\dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{15}}{5}$　　$\therefore \dfrac{\sqrt{3}}{5} < \dfrac{3}{5} < \sqrt{\dfrac{3}{5}} < \dfrac{3}{\sqrt{5}}$

② $7\sqrt{6} = \sqrt{294}$，$6\sqrt{7} = \sqrt{252}$，$16 = \sqrt{256}$

　　$\therefore 6\sqrt{7} < 16 < 7\sqrt{6}$

③ $\left(\sqrt{10+2}\right)^2 = \left(\sqrt{12}\right)^2 = 12$

　$\left(\sqrt{10} + \sqrt{2}\right)^2 = 10 + 4\sqrt{5} + 2 = 12 + 4\sqrt{5}$

　$\therefore \sqrt{10+2} < \sqrt{10} + \sqrt{2}$

④ $\left(3 + \sqrt{3}\right)^2 = 9 + 6\sqrt{3} + 3 = 12 + 6\sqrt{3} \simeq 22.392$

　$\left(\sqrt{5} + \sqrt{7}\right)^2 = 5 + 2\sqrt{35} + 7 = 12 + 2\sqrt{35} \simeq 23.832$

　　$\therefore 3 + \sqrt{3} < \sqrt{5} + \sqrt{7}$

⑤ $(\sqrt{5} - 1)(\sqrt{5} + 1) = (\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})$

　　　　　　　$= (\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3}) = 4$

　又 $\sqrt{5} + 1 < \sqrt{6} + \sqrt{2} < \sqrt{7} + \sqrt{3}$

　$\therefore \sqrt{5} - 1 > \sqrt{6} - \sqrt{2} > \sqrt{7} - \sqrt{3}$

立即練習五

p. 65(1)

　　$\sqrt{8^2 + 15^2} = 17$　　$17 + 8 = 25$

p. 65(2)

① $\overline{AH} = \sqrt{10^2 - 6^2} = 8$　　$\overline{AC} = \sqrt{8^2 + 15^2} = 17$

　$\therefore \triangle ABC$ 周長 $= 10 + 17 + 21$

②$\overline{HF} = \dfrac{\overline{AH} \times \overline{CH}}{\overline{AC}} = \dfrac{8 \times 15}{17}$

p. 65(3)

正方形 ACQP = 正方形 ABDE + 正方形 BCFG

$$= 4(\Delta BOD + \Delta GSF) = 4 \times 24 = 96 \ \left(平方單位\right)$$

p. 65(4)

①甲、乙面積和 $= \overline{BC}^2 + \overline{CD}^2 = \overline{BD}^2 = \left(\sqrt{120}\right)^2$

$$= 120 \quad \left(平方公分\right)$$

②設$\overline{BC} = a$，$\overline{CD} = b$，則$(a + b) \times 2 = 28 \Rightarrow a + b = 14$

又$a^2 + b^2 = \overline{BD} = 120$

∴ $2ab = (a + b)^2 - (a^2 + b^2) = 196 - 120 = 76 \Rightarrow ab = 38$

故長方形 ABCD 面積 $= ab = 38 \quad \left(平方公分\right)$

p. 65(5)

$\overline{FJ} = \sqrt{12^2 + (12 - 7)^2} = 13$

立即練習三

p. 81(1)

①原式 $= x(a - b + c) - y(a - b + c) = (x - y)(a - b + c)$

②原式 $= a(ac - bd - bc + ad) = a[c(a - b) + d(a - b)]$

$$= a\,(a - b)(c + d)$$

p. 81(2)

①原式 $= 2xy - 4 - 8x + y = 2x(y - 4) + (y - 4)$

$$= (y - 4)(2x + 1)$$

②原式 $= ax^3 + ax^2 + bx^2 + ax + bx + a$

$$= a(x^3 + x^2 + x + 1) + bx(x + 1)$$

$$= (x + 1)(ax^2 + a) + bx(x + 1)$$
$$= (x + 1)(ax^2 + bx + a)$$

③原式 $= abx^2 + aby^2 - a^2xy - b^2xy$

$$= (ax)(bx) + (ay)(by) - (ax)(ay) - (bx)(by)$$
$$= bx(ax - by) - ay(ax - by)$$
$$= (ax - by)(bx - ay)$$

立即練習四

p. 88(1)

$$原式 = (x - 1)(2x + 3)[(2x + 3)^2 - (x - 1)^2]$$

$$= (x - 1)(2x + 3)(3x + 2)(x + 4)$$

p. 88(2)

①原式 $= 4 - 4b^2 - 4ab - a^2 = 2^2 - (a + 2b)^2$

$$= (2 + a + 2b)(2 - a - 2b)$$

②原式 $= a^2b^2 - a^2 - b^2 + 1 - 4ab = (ab - 1)^2 - (a + b)^2$

$$= (ab - 1 + a + b)(ab - 1 - a - b)$$

p. 88(3)

①原式 $= x^2(x^2 + y^2) + y^2(x^2 + y^2) - x^2y^2$

$$= (x^2 + y^2)(x^2 + y^2) - x^2y^2$$
$$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$$

②原式 $= (x^2 + 3) - x^2 = (x^2 + x + 3)(x^2 - x + 3)$

立即練習三

p. 95(1)

$$原式 = (7x - 2y)^3 - y^2(7x - 2y)$$

$$= (7x - 2y)[(7x - 2y)^2 - y^2]$$
$$= (7x - 2y)[(7x - 2y + y)(7x - 2y - y)]$$
$$= (7x - 2y)(7x - y)(7x - 3y)$$

p. 95(2)

$$2x^2 + 3bx + 2x + 3b = 3b(x + 1) + 2x(x + 1)$$
$$= (3b + 2x)(x + 1)$$

p.95(3)

$A = 2x^2 - 5x + 2 = (2x - 1)(x - 2)$
$B = x^2 + x - 6 = (x + 3)(x - 2)$
$C = 2x^2 + 5x - 3 = (2x - 1)(x + 3)$

∴ 三邊長為 $2x - 1$、$x - 2$、$x + 3$

p.95(4)

原式 $= \dfrac{(x^2 + 2 + 2x + 3)(x^2 + 2 - 2x - 3)}{x^2 + 2x + 5} = x^2 - 2x - 1$

∴ $a + b = -2 - 1 = -3$

p.95(5)

$x^2 + 2x - 3 = (x + 3)(x - 1)$
$(x + 3) \Rightarrow (x + 3)(x - 2) = x^2 + x - 6$
$(x - 1) \Rightarrow (x - 1)(x + 6) = x^2 + 5x - 6$

∴ $a = 1$ 或 5

立即練習三

p.110(1)

①$x^2 + 4x + 4 - 3x - 6 - 10 = 0$

$x^2 + x - 12 = 0 \Rightarrow (x - 3)(x + 4) = 0$

∴ 兩根為 3 或 -4

②將 $x = -4$ 代入$x^2 + kx + 3k = 0$

得 $16 - 4k + 3k = 0 \Rightarrow k = 16$

$x^2 + 16x + 48 = 0 \Rightarrow (x + 4)(x + 12) = 0$

$\Rightarrow x = -4$ 或 -12 ∴ 方程式乙的另一根為 -12

p.110(2)

①$3x^2 - 3a + ax - 9x = 3x(x - 3) + a(x - 3) = 0$

$\Rightarrow (x - 3)(3x + a) = 0$ ∴ $x = 3$ 或 $-\dfrac{a}{3}$

②$3(x + 2)^2 + 5(x + 2)(2x - 1) - 2(2x - 1)^2 = 0$

$$\Rightarrow [(x+2)+2(2x-1)][3(x+2)-(2x-1)]=0$$
$$\Rightarrow (x+2+4x-2)(3x+6-2x+1)=0$$

$$\Rightarrow 5x(x-7)=0 \quad \therefore x=0 \text{ 或 } -7$$

p.110(3)

　　①小天：$(x-3)(x-7)=0 \Rightarrow x^2-10x+21=0$

　　　佑廷：$(x-2)(x+12)=0 \Rightarrow x^2+10x-24=0$

　　　　$\therefore$ 正確的方程式為$x^2+10x+21=0$

　　②$x^2+10x+21=0 \Rightarrow (x+3)(x+7)=0$

　　　　$\Rightarrow x=-3 \text{ 或 } x=-7$

p.110(4)

　　$8(x+3)(x-5)=7x(x-2) \Rightarrow x^2-2x-120=0$

　　　　$\Rightarrow (x+10)(x-12)=0 \Rightarrow x=-10 \text{ 或 } 12$

　　$\because \alpha > \beta \Rightarrow \alpha=12 , \beta=-10 \quad \therefore \alpha+2\beta=-8$

立即練習六

p.124(1)

　　$\because (a \ast b)=(a+1)(2b-1)-3$

　　$\Rightarrow [(x+1)(2x+1)-3] \ast 1 = -3$

　　$\Rightarrow (2x^2+x-4) \ast 1 = -3$

　　$\Rightarrow (2x^2+x-4+1)(2 \times 1-1)-3 = -3$
　　$\Rightarrow (2x^2+x-3)(1)-3 = -3$

　　　$\Rightarrow 2x^2+x-6-3=0 \quad x=1 \text{ 或 } -\dfrac{3}{2}$

p.124(2)

　　$ax^2+bx+c=0$，設兩根為 α、β

$$\alpha + \beta = -\frac{b}{a}，\alpha \times \beta = \frac{c}{a} = 1 \qquad \therefore a = c$$

p.124(3)

$$x^2 + 6x - 5616 = 0 \Rightarrow x^2 + 6x + 9 = 5616 + 9$$

$$\Rightarrow (x + 3)^2 = 5625 \Rightarrow x + 3 = \pm 75 \Rightarrow x = 72 \text{ 或} - 78$$

p.124(4)

$$D = b^2 - 4ac = (-98)^2 - 4(49)(-1) = 9800$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{98 \pm \sqrt{9800}}{98} \simeq \frac{98 \pm 98.99}{98} \qquad \left(\text{一正一負}\right)$$

立即練習三

p.132(1)

設正方形原邊長為 x

$$2(x - 4)^2 = 72 \qquad \therefore x = 10 \text{ 或} - 2\left(\text{不合}\right)$$

p.132(2)

志翔所看到的方程式$(x + 2)(x + 3) = 0 \Rightarrow x^2 + 5x + 6 = 0$

正確方程式$x^2 + 5x - 6 = 0$，$(x + 6)(x - 1) = 0 \qquad x = -6 \text{ 或 } 1$

p.132(3)

$\because$ 兩根互為相反數 $\therefore$ 兩根和為 0

$\therefore$ 一次項係數為 0，故$a^2 - 4 = 0$，$\therefore a = \pm 2$

$a = -2$ 代入方程式為 $2x^2 = 0 \qquad \therefore x = 0$

$a = 2$ 代入方程式為 $2x^2 - 4 = 0 \qquad \therefore x^2 = 2$，$x = \pm\sqrt{2}$